Designing a banking system to eliminate the potential for catastrophe

Kentaro Asai

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Abstract

This paper investigates how bank risk-taking interacts with deposit market structure, bank transparency, deposit insurance, and the incentive structure of executive bankers. Unlike previous studies, our model endogenizes both the portfolio choice and the default decision of banks. As a result, the banking sector can attain multiple equilibria, often including one that induces high default risk or risk-shifting. Although direct asset restrictions or market concentration can eliminate the potential for an unfavorable equilibrium, these policies harm social welfare. Public disclosure of banks' risk can deregulate these policies; however, the favorable impact of transparent banking is offset by deposit insurance. Instead, debt-type managerial compensation eliminates risk-shifting. Moreover, it removes the potential for high default risk in an economic downturn by simultaneously providing liquidity to banks during crises. Surprisingly, this alternative scheme neither sacrifices social welfare nor the coverage of deposit insurance. Policy packages based on the calibrated model are also proposed.

1 Introduction

Excessive financial risk-taking has played a significant role in the global financial crisis (GFC). In the aftermath of the crisis, as part of the Dodd Frank Act (DFA), US regulators increased disclosure and transparency requirements for the banking business and enhanced the compensation oversight of the financial industry. The Capital Requirements Directive IV (CRD IV), the EU's new legislative package, aims to establish liquidity requirements and revise the incentive structure of executive bankers. Simultaneously, to prevent bank runs, the US raised the threshold on deposit insurance and eventually covered all non-interest-bearing transaction accounts. Similarly, the EU raised the threshold on deposit insurance. Although the de facto antitrust exemption for banking has been eroding in the US and the EU for the last decades[1], both systems took measures against competition during the crisis: the US arranged mergers between Bear Stearns and JP Morgan and between Merrill Lynch and Bank of America, whereas the UK arranged a merger between Lloyds TBC and HBOS.

To avoid another financial crisis, regulators are required to understand the forces that affect the risk-taking of banks and examine what policies can efficiently prevent the excessive risk-taking of banks. The purpose of this paper is to analyze the risk-taking behavior of banks and identify a set of policies that improves the soundness of the banking sector without harming social welfare.

Previous studies have investigated the relationship between the soundness of the financial sector and direct asset restrictions[2] [Nicoló et al., 2012, Goodhart et al., 2012], deposit market competition [Matutes and Vives, 2000, Allen and Gale, 2004, Boyd and Nicoló, 2005, Martinez-Miera and Repullo, 2010], public disclosure of banks' risk [Cordella and Yeyati, 1998], deposit insurance [Diamond and Dybvig, 1983], and compensation structure [Bolton et al., 2011, DeMarzo et al., 2014]. According to these studies, direct asset restrictions are effective for the reduction of default risk while they significantly reduce social welfare. The correlation between risk-taking and deposit market competition can be both positive and negative. Public disclosure of banks' risk reduces default risk but this is offset by deposit insurance, which prevents bank runs. Debt-like compensation for executives reduces risk for financial institutions.

However, few papers have endogenized both the portfolio choice and the default decision of banks. Because of the limited strategic options available to banks, they might have missed the equilibria that potentially could emerge. As a result, they have fewer equilibria than this paper does. Diamond and Dybvig [1983] suggested that the soundness of the banking sector changes when switching from one equilibrium to another much more than by the perturbation within a single equilibrium. As shown later, such regime-switching can raise the probability of default from negligible to catastrophic. Moreover, it can change

[1] See OECD [2009].

[2] In practice asset restrictions relate to capital asset ratios, with risk-weighted measures of assets, and limitations on "large exposures" and the concentration of risks [Matutes and Vives, 2000].

the level of risky asset exposure from minimum to maximum. Although the first priority of regulators is to eliminate the potential for an unfavorable equilibrium to exist, few studies have analyzed the interaction of multiple policies in the context of multiple equilibria. As an exception, Egan et al. [2014] has investigated this issue, but these authors did not endogenize the portfolio choice of banks. Therefore, the implication for potential excessive risky asset exposure (risk-shifting) still remains unknown. In addition, although previous literature has addressed the interaction between deposit market competition, bank transparency, and deposit insurance as well as bank transparency, deposit insurance, and compensation regulations, it has not analyzed the interaction between deposit market competition and compensation regulations.

To the best of our knowledge, this paper is a first attempt to endogenize both the portfolio choice and the default decision of banks, taking into account strategic interactions among stakeholders. This enables us to analyze multiple equilibria, including the one that induces high default risk or risk-shifting, and propose a scheme that eliminates the potential for an unfavorable equilibrium. Moreover, our framework allows regulators to simultaneously analyze the policies from multiple domains, including antitrust policies and compensation regulations, in an integrated way.

Our model suggests that direct asset restrictions or market concentration can reduce the potential for equilibria to exist in either high default risk or risk-shifting. Nevertheless, these policies harm social welfare. Conversely, disclosing the risks that banks pose to depositors leads to relaxation of the restrictions on the level of risky loans and the entries to deposit market market, which is necessary to prevent catastrophic consequences. However, the favorable impact of bank transparency is offset by deposit insurance. Instead, debt-type managerial compensation can eliminate risk-shifting without harming social welfare or shrinking the coverage of deposit insurance. This is because market competition is complementary to the implementation of debt-type managerial compensation. In addition, it can eliminate the potential for a high-risk equilibrium to exist in an economic downturn if regulators simultaneously provide sufficient liquidity to banks during crises.

According to our calibration, a catastrophic equilibrium may emerge under the current banking system without regulations. We propose policy packages based on the calibrated model.

The rest of this paper is structured as follows. Section 2 describes the benchmark model. Section 3 solves the equilibria of the benchmark model. Section 4 analyzes the impact of transparent banking. Section 5 characterizes the optimal tax on the CDS spread of a bank. Section 6 provides quantitative implications from the calibrated model. Section 7 discusses policy recommendations.

2 Model

In this section, we describe our benchmark model. We assume the same funding structure of banks, as that in Egan et al. [2014]. Banks are financed through deposits, which have to be repaid at the end of each period. The residual claimants are “deep pocket” equity holders, as that in Leland [1994]. If there is a shortfall, then the equity holders can decide whether to inject enough funds to repay the deposits or to default. In the case of a default, the equity holders are protected by limited liability. At bankruptcy, the bank is sold and the proceeds are used to repay the depositors.

2.1 Players

There are n identical banks ($n \geq 2$) that compete with each other to collect funds from the same pool of depositors. There is no outside option that the depositors can avail in the absence of banks. Each bank is run by a risk-neutral manager, who is hired by shareholders under a linear incentive contract[3] that depends on the bank's stock price. In other words, managerial incentives are perfectly aligned with the interests of the shareholders[4].

All depositors consider interest rates when they choose their banks. Some depositors are not insured by the deposit insurance authority; thus, their preference is also sensitive to the default risk of each bank. On the other hand, the insured depositors do not consider banks' default risk because any shortfall is compensated by the deposit insurance authority. As insured households observe all the information they need, they are excluded from this game. Instead, the deposit insurance authority requires a bank to pay an actuarially fair premium; thus its strategy matters to the shareholders of the bank.

Consequently, the players of this game are (1) banks (managerial incentives aligned with the interests of shareholders); (2) the deposit insurance authority; and (3) the uninsured depositors. Each bank simultaneously chooses its risk profile. The deposit insurance authority sets the premium for each bank without observing the riskiness of each bank. The uninsured depositors choose their banks without observing the default risk of each bank. We denote the strategy of the manager at bank k by σ_k and the strategies of other players by σ_{-k}.

2.2 Timing

The timing of the benchmark game is similar to that of Bolton et al. [2011].

1. For each bank k, the incumbent equity holders hire a manager under an incentive contract:

[3] For example, a manager receives a fixed salary at the beginning of the year and an equity-linked bonus at the end of the year. Linear contracts are common in practice [Bose et al., 2011].

[4] Later, we will relax this assumption and consider the situation where managerial incentives are not perfectly aligned with the interests of shareholders.

$W(\sigma_k, \sigma_{-k}) = W_0 + \delta^E V(\sigma_k, \sigma_{-k})$, where W_0 is a fixed wage, δ^E is the shares of equity ($\delta^E > 0$), and $V(\sigma_k, \sigma_{-k})$ is the equity value of the bank. The manager of the bank immediately receives W_0.

2. The deposit insurance authority sets the vector of actuarially fair premiums $\xi P(\boldsymbol{\sigma^I})$, where $P(\boldsymbol{\sigma^I})$ is the vector of the default rates, $\boldsymbol{\sigma^I}$ is the deposit insurance authority's belief in the strategies of all managers, and ξ is a fire-sale discount rate ($0 < \xi \leq 1$).

3. Each bank simultaneously chooses σ_k. The bank also determines deposit rates and announces them to the depositors.

4. The depositors are informed about the deposit rates of banks, but they do not observe the strategies of banks. Instead, they believe $\boldsymbol{\sigma^N}$ for the strategies of banks and choose their banks. They expect the vector of the banks' default risk to be $P(\boldsymbol{\sigma^N})$. Each bank acquires deposits from the same pool of insured and uninsured depositors, M^I and M^N, respectively ($M^I + M^N = 1$, $M^I \geq 0$, and $M^N \geq 0$).

5. $V(\sigma_k, \sigma_{-k})$ is determined by an efficient stock market. Each bank pays a bonus to the manager in accordance with the incentive contract.

6. The return on the bank assets, $\tilde{R}_k$, is realized. After paying the premium to the deposit insurance authority and interest to the depositors, the shareholders make a default decision. In the case of a default, the bank assets can be sold at a discount. As ξ is the fire-sale discount rate, the depositors recover $(1-\xi)$ of their claims. However, the deposit insurance authority compensates the insured depositors for any shortfall.

2.3 Banks' strategies

Each bank k determines (1) portfolio choice; (2) default decision; (3) insured deposit rate; and (4) uninsured deposit rate. Because (3) and (4) can be uniquely determined by (1) and (2), we can eventually reduce the strategy space of the bank to that of (1) and (2).

For (1), we allow the manager to invest in either risky loans or riskless bonds. Let q_k be the exposure to risky loans ($q_k > 0$). Then, the manager invests $1 - q_k$ of the bank assets into riskless bonds. Therefore, we can represent the return on the bank assets as $\tilde{R}_k = (1-q_k)\mu_0 + q_k\tilde{R}$, where μ_0 is the risk-free rate and $\tilde{R}$ is the return on risky loans. We assume that the return on risky loans follows a normal distribution, $\tilde{R} \sim N[\mu, \nu]$, where $\nu > 0$. Without loss of generality, we can set $\mu_0 = 0$ by measuring each rate relative to the risk-free rate. Throughout the paper, we denote $\Phi(.), \phi(.), \lambda(.)$ as the CDF, PDF, and inverse Mills Ratio of standard normal distribution, respectively.

Regarding (2), Hortaçsu et al. [2011] showed that shareholders plan the reservation rate to make a default decision. Let the reservation rate be $q_k R_k > 0$, such that the bank continues to operate if $q_k\tilde{R} > q_k R_k$, otherwise, it liquidates its assets. Then, the probability of default is $\Phi(\frac{R_k-\mu}{\nu})$. Let the normalized reservation rate be z_k, where $z_k = \frac{R_k-\mu}{\nu}$.

We represent the default decision of the bank by z_k because it sufficiently represents the probability of default.

Consequently, the bank optimizes both the overall risk q_k (portfolio choice) and tail risk z_k (default decision). Then, its strategy is represented by σ_k where $\sigma = (q_k, z_k)$. We also denote the strategies of other players by $\sigma_{-k} = \{\{\sigma_{k'}\}_{k' \neq k}, \boldsymbol{\sigma^I}, \boldsymbol{\sigma^N}\}$.

2.4 Depositors' strategies

We model the demand for deposits in a discrete choice framework. As the insured depositors are protected by deposit insurance, they only care about deposit rates when choosing a bank for opening accounts. On the other hand, the uninsured depositors are not protected by the deposit insurance; therefore, they also consider the default risk of banks besides the deposit rates[5]. Household j derives indirect utility from holding insured and uninsured deposits at bank k, where

$$\tilde{u}_j^I(i_k) = \alpha i_k + \tilde{\epsilon_{j,k}}$$

$$\tilde{u_j}^N(i_k, \boldsymbol{\sigma^N}) = \alpha(i_k - \xi P_k(\boldsymbol{\sigma^N})) + \tilde{\epsilon_{j,k}}.$$

Here i_k and $P_k(\boldsymbol{\sigma^N})$ represent the deposit rate and the probability of default associated with bank k, respectively. The parameter α measures depositors' effective deposit rate sensitivity, which is the total expected return on a depositor's claim, taking into account the default risk of a bank. For example, if the deposit rate is 10%, the probability of a default is 5%, and the fire-sale discount rate is 50%, the uninsured depositor expects to gain 10 dollars and lose 5 dollars. Therefore, the total expected return on the uninsured depositor's claim is 5 dollars. Then, the effective deposit rate is 5%.

Further, $\tilde{\epsilon_{j,k}}$ is the consumer's idiosyncratic utility shock which follows a iid Type 1 Extreme Value distribution. Assuming that there are infinitely many depositors, bank k acquires market shares in insured and uninsured deposit markets, where

$$s^I(i_k, \sigma_{-k}) = \frac{\exp(\alpha i_k)}{\sum_{k'=1}^{n} \exp(\alpha i_{k'})}$$

$$s^N(i_k, \sigma_{-k}) = \frac{\exp(\alpha i_k - \gamma \Phi(z_k^N))}{\sum_{k'=1}^{n} \exp(\alpha i_{k'} - \gamma \Phi(z_{k'}^N))}.$$

The uninsured depositors become worse off if their belief regarding the strategy of each bank differs from the truth. They might choose the wrong bank, which would not be best for them. Thus, the optimal strategy of the uninsured depositors is to set their belief regarding the strategy of each bank identical to the actual one.

[5] Egan et al. [2014] empirically rejected that the demand for insured deposits is sensitive to the banks' default risks, while the uninsured depositors care about them.

2.5 Deposit insurance authority's strategy

As the deposit insurance authority attempts to achieve actuarially fair insurance, it sets the premium equal to the probability of default multiplied by the fire-sale discount ξ for each bank k as follows:

$$\xi P_k(\boldsymbol{\sigma^I}) = \xi \Phi(z_k^I).$$

We assume that the deposit insurance authority is strictly worse off when it sets the premium of each bank either strictly above or below the expected payment to the insured depositors of the bank. Thus, the authority's optimal strategy is to set their belief regarding the strategy of each bank identical to the actual one, like the uninsured depositors.

3 Equilibrium

In this section, we solve the equilibrium of the benchmark game and derive policy implications from our results. Our analysis predicts the potential for a high-risk or risk-shifting equilibrium without regulation, the need for direct asset restrictions or market concentration to prevent the potential for such equilibrium, and the negative side effects of these policies on social welfare.

3.1 Symmetric local Nash equilibrium

Our goal is to characterize the symmetric local Nash equilibrium. We denote the game described above by Γ. We assume symmetry to obtain the analytical closed form expressions, which are particularly attractive to regulators because of their simplicity. A local Nash equilibrium is a weaker equilibrium concept, but it is still robust to the local perturbation of each player's strategy.

Let the strategy space of the bank be $\Sigma = Q \times \mathbb{R}$, where Q is the closed interval $[\underline{q}, \bar{q}]$, $\underline{q} > 0$, and $\bar{q} > \underline{q}$. Then, the strategy space for all banks is Σ^n and the strategy space of the deposit insurance authority is Σ^n, which is the space for all banks' strategies. Similarly, the strategy space of the uninsured depositors is Σ^n. Thus, the strategy space for all players in this game is $\Sigma^n \times \Sigma^n \times \Sigma^n$. Extending the definition by Ratliff et al. [2013] for $n+2$ players, we define the symmetric local Nash equilibrium as the following.

Definition 1. A strategy $\{\{\sigma_k\}_{k=1}^n, \boldsymbol{\sigma^N}, \boldsymbol{\sigma^I}\} \in \Sigma^n \times \Sigma^n \times \Sigma^n$ is a symmetric local Nash equilibrium of Γ if $\sigma_k = \sigma, \forall k = 1, ..., n$, and either
(i) there exist open rectangles $W \subset \Sigma, \forall k = 1, ..., n$, such that
$\sigma \in W$, $V(\sigma, \{\{\sigma\}_{k' \neq k}, \boldsymbol{\sigma^N}, \boldsymbol{\sigma^I}\}) \geq V(\sigma', \{\{\sigma\}_{k' \neq k}, \boldsymbol{\sigma^N}, \boldsymbol{\sigma^I}\})$, $\forall \sigma' \in W \setminus \sigma$, $\boldsymbol{\sigma^N} = \boldsymbol{\sigma^I} = \boldsymbol{\sigma}$, and $q \in (\underline{q}, \bar{q})$, or
(ii) there exist half-open rectangles, i.e. the cartesian products of right (left) half-open interval and open interval, $W \subset \Sigma$, $\forall k = 1, ..., n$, such that

$\sigma \in W$, $V(\sigma, \{\{\sigma\}_{k' \neq k}\boldsymbol{\sigma^N}, \boldsymbol{\sigma^I}\}) \geq V(\sigma', \{\{\sigma\}_{k' \neq k}\boldsymbol{\sigma^N}, \boldsymbol{\sigma^I}\})$, $\forall \sigma' \in W \setminus \sigma$, and $\boldsymbol{\sigma^N} = \boldsymbol{\sigma^I} = \boldsymbol{\sigma}$, and $q = \underline{q}(\bar{q})$.

3.2 Deposit rates

First, we characterize the optimal deposit rates chosen by the bank ($i^I(\sigma_k, \sigma_{-k}), i^N(\sigma_k, \sigma_{-k})$). At the optimum, the bank makes the expected markup equal to the inverse price elasticity of the residual demand as follows:

$$q_k[\mu + \nu\lambda(z_k)] - \xi\Phi(z_k^I) - i^I(\sigma_k, \sigma_{-k}) = \frac{1}{\alpha[1 - s^I(\sigma_k, \sigma_{-k})]} \tag{1}$$

$$q_k[\mu + \nu\lambda(z_k)] - i^N(\sigma_k, \sigma_{-k}) = \frac{1}{\alpha[1 - s^N(\sigma_k, \sigma_{-k})]}, \tag{2}$$

where $s^I(\sigma_k, \sigma_{-k}) = s^I(i^I(\sigma_k, \sigma_{-k}), \sigma_{-k})$, $s^N(\sigma_k, \sigma_{-k}) = s^N(i^N(\sigma_k, \sigma_{-k}), \sigma_{-k})$.

The LHS of (1) and (2) are strictly decreasing in deposit rate; whereas the RHS of (1) and (2) are strictly increasing in it. Moreover, the RHS of (1) and (2) approach positive infinity as the deposit rate approaches positive infinity, whereas they converge to $\frac{1}{\alpha}$ as the deposit rate approaches negative infinity. Furthermore, the LHS of (1) and (2) approach negative infinity as the deposit rate approaches positive infinity; whereas they approach positive infinity as the deposit rate approaches negative infinity. These facts suggest that, for a given σ_k, optimal deposit rates uniquely exist.

Lemma 1. For any (σ_k, σ_{-k}), $i^I(\sigma_k, \sigma_{-k})$ and $i^N(\sigma_k, \sigma_{-k})$ satisfying (1) and (2) uniquely exist.

3.3 Valuation of equity

The expected return and profit of the bank is characterized as the weighted average of the markups extracted from the insured and uninsured depositors as follows:

$$\pi(\sigma_k, \sigma_{-k}) = \frac{\theta^I(\sigma_k, \sigma_{-k})}{\alpha[1 - s^I(\sigma_k, \sigma_{-k})]} + \frac{1 - \theta^I(\sigma_k, \sigma_{-k})}{\alpha[1 - s^N(\sigma_k, \sigma_{-k})]}.$$

Here $\theta^I(\sigma_k, \sigma_{-k})$ is the weight of insured deposits in the bank liability. In accordance with this, we also characterize the expected profit of the bank as follows:

$$\Pi(\sigma_k, \sigma_{-k}) = \frac{M^I s^I(\sigma_k, \sigma_{-k})}{\alpha[1 - s^I(\sigma_k, \sigma_{-k})]} + \frac{(1 - M^I)s^N(\sigma_k, \sigma_{-k})}{\alpha[1 - s^N(\sigma_k, \sigma_{-k})]}.$$

Given the reservation rate strategy, we can characterize the equity value as the expected profit of the bank multiplied by the survival probability discounted by the risk-adjusted rate, i.e. the sum of the normal discount rate, r $(0 < r \leq 0.15)$[6], and the default risk of the bank:

$$V(\sigma_k, \sigma_{-k}) = \frac{(1 - \Phi(z_k))\Pi(\sigma_k, \sigma_{-k})}{r + \Phi(z_k)}.$$

3.4 Default decision

In general, the marginal value of taking tail risks is[7] as follows:

$$\begin{aligned}\frac{\partial V(\sigma_k, \sigma_{-k})}{\partial z_k} = &- \frac{\left(M^I s^I(\sigma_k, \sigma_{-k}) + (1 - M^I) s^N(\sigma_k, \sigma_{-k})\right)}{r + \Phi(z_k)} \\ &\times \left\{ \ \phi(z_k) \left(\tfrac{1+r}{r+\Phi(z)} \pi(\sigma_k, \sigma_{-k}) - q_k \nu(\lambda(z_k) - z_k) \right) \ \right\}. \end{aligned} \tag{3}$$

At equilibrium, this is simplified to the following:

$$\frac{1+r}{r + \Phi(z)} \frac{n}{\alpha(n-1)} - q\nu \left[\lambda(z) - z\right] = 0. \tag{4}$$

Here (4) implies that the going concern value of the bank has to be equal to the shortfall needed for the bank to continue business when the reservation rate is realized. If the going concern value of the bank is lower (higher) than the shortfall at the threshold, the shareholders are unwilling (willing) to inject capital even if the realized return is slightly above (below) the threshold. Therefore, at the threshold, the shareholders have to be indifferent between stopping and continuing bank business.

Rewriting (4), we have

$$\frac{n}{\alpha(n-1)q\nu} = \frac{r + \Phi(z)}{1 + r} \left[\lambda(z) - z\right]. \tag{5}$$

The LHS of (5) is the return on the bank equity divided by the standard deviation of the portfolio return, namely, the Sharpe ratio of the bank equity[8]. Moreover, the RHS of (5) is the cost of capital. Note that the return required by the shareholders is not a function of exposure to risky loans. In other words, we can separate the return required by the shareholders from the optimal portfolio choice. This enables a simple characterization of the equilibrium of the game.

We denote the Sharpe ratio of the bank equity by $g(q, n)$, which decreases in the exposure to risky loans and the number of banks in the deposit market because market

[6] Our model requires r to be reasonably low to have meaningful implications.

[7] Hortaçsu et al. [2011] showed that the optimal reservation rate is the root of the last term in (3). This is exactly the first-order condition for the optimal tail risk.

[8] Note that every return is relative to the risk free rate in our model.

competition reduces the markup earned by the bank, whereas increasing the exposure to risky loans raises the standard deviation of the portfolio return. We also denote the RHS of (5) by $h(z)$. As $h(z)$ is not affected by any policy parameter, the Sharpe ratio of the bank equity determines the tail risk at equilibrium. As $g(q, n)$ is not affected by μ, the default risk of each bank is insensitive to μ as long as q is fixed.

If a slight decrease in the reservation rate satisfying (5) makes the going concern value of the bank larger than the required capital injection, then shareholders further decrease the reservation rate; otherwise, they would irrationally liquidate the bank assets even if it is higher than the required capital injection. Moreover, if a slight increase in the reservation rate satisfying (5) makes the going concern value smaller than the required capital injection, shareholders further increase the reservation rate; otherwise, they would irrationally continue bank business even if the value of continuing business is lower than the shortfall. We find that the locally stable reservation rate requires the following condition:

$$\frac{\lambda'(z)}{n} - h'(z) \geq 0. \tag{6}$$

Then, the sufficient condition for local stability is

$$\frac{\lambda'(z)}{n} - h'(z) > 0. \tag{7}$$

3.5 Financial stability

When the discount rate is modestly low[9], $h(z)$ decreases in z for $z < z^1$, attains a local minimum at $z = z^1$, strictly increases in z for $z^1 < z < z^2$, attains a local maximum at $z = z^2$, and strictly decreases in z for $z > z^2$. We can confirm this from Figure 1. Moreover, we define $\underline{z} = \inf\{z | g(\underline{q}, n) = h(z)\}$ and $\bar{z} = \sup\{z | g(\bar{q}, n) = h(z)\}$. We categorize the equilibrium based on the performance of financial stability as follows.

Definition 2. An equilibrium strategy σ is high-risk if $z \geq z^2$, low-risk if $z \leq z^1$, and middle-risk if $z^1 < z < z^2$.

We notice that $h'(z) \leq 0$ if $z \leq z^1$ and $z \geq z^2$ while $h'(z) > 0$ if $z^1 < z < z^2$. Then, we claim the following.

Lemma 2. If $z \leq z^1$ or $z \geq z^2$, then z satisfies (7).

For reference, we document the critical values for moderately low r in the table given below. The default risk of each bank is at least more than half for a high-risk equilibrium, whereas it is, at most, 0.04 for a low-risk equilibrium.

[9] We verify that this holds for $0 < r \leq 0.15$.

r	0.01	0.05	0.1	0.15
$\Phi(z^1)$	0.00097	0.0082	0.023	0.036
$\Phi(z^2)$	0.76	0.76	0.73	0.69

3.6 Portfolio choice

Next, we consider the optimal portfolio choice. In general, the marginal value of the exposure to risky loans is

$$\frac{\partial V(\sigma_k, \sigma_{-k})}{\partial q_k} = \frac{1-\Phi(z_k)}{r+\Phi(z_k)} \left(M^I s^I(\sigma_k, \sigma_{-k}) + (1-M^I) s^N(\sigma_k, \sigma_{-k}) \right) (\mu + \nu\lambda(z_k)). \tag{8}$$

At equilibrium, this is simplified to

$$\frac{1-\Phi(z)}{r+\Phi(z)} \frac{\mu + \nu\lambda(z)}{n}. \tag{9}$$

Note that the marginal value of the overall risk is positive if $\mu > -\nu\lambda(z)$, neutral if $\mu = -\nu\lambda(z)$, and negative if $\mu < -\nu\lambda(z)$. Therefore, if the expected return on risky loans is greater than the risk-free rate, the marginal value is always positive. Moreover, even if the expected return is smaller than the risk-free rate, the marginal value of the exposure to risky loans can be positive because the bank manager benefits from the upside of the unprofitable gamble without incurring its downside because of limited liability.

Lemma 3. Suppose σ is an equilibrium strategy. $q = \bar{q}$ if $\mu > -\nu\lambda(z)$ and $q = \underline{q}$ if $\mu < -\nu\lambda(z)$. There is no locally stable q if $\mu = -\nu\lambda(z)$.

Proof. This is obvious from (9). While q is arbitrary if $\mu = -\nu\lambda(z)$, it is not robust to local perturbation. If $q \in [\underline{q}, \bar{q})$, a slight increase in z makes $\mu > -\nu\lambda(z+\epsilon)$, and hence q jumps up to $\bar{q}$. Moreover, if $q = \bar{q}$, then a slight decrease in z makes $\mu < -\nu\lambda(z-\epsilon)$, and hence q jumps down to $\underline{q}$.

□

3.7 Credit control

We also categorize the equilibrium based on the performance of credit control. We assume that a society is risk-neutral because its portfolio is well-diversified. It then wants a bank to undergo the largest exposure to risky loans, as long as the expected return exceeds the risk-free rate, but the least exposure when the expected return is below the risk-free rate. Therefore, we evaluate the equilibrium based on banks' credit control as follows.

Definition 3. An equilibrium strategy σ is underinvesting if $q = \underline{q}$ and $\mu > 0$ and risk-shifting if $q = \bar{q}$ and $\mu < 0$. The strategy can be termed as optimally credit-controlling if it is neither underinvesting nor risk-shifting.

3.8 Results

Proposition 1. There exists at least one symmetric local Nash equilibrium under Γ. If σ is a symmetric local Nash equilibrium strategy of Γ, σ satisfies (5), (6) and either $q = \bar{q} \wedge \mu > -\nu\lambda(z)$ or $q = \underline{q} \wedge \mu < -\nu\lambda(z)$. Conversely, if σ satisfies (5), (7), and either $q = \bar{q} \wedge \mu > -\nu\lambda(z)$ or $q = \underline{q} \wedge \mu < -\nu\lambda(z)$, σ is a symmetric local Nash equilibrium strategy of Γ.

Proof. If $\mu \geq -\nu\lambda(\underline{z})$, then $\sigma = (\bar{q}, \bar{z})$ is an equilibrium because $\bar{z}$ is on the low or high domain, which is locally stable according to Lemma 2, and $\mu + \nu\lambda(z) > \mu + \nu\lambda(\underline{z}) \geq 0$.

If $\mu < -\nu\lambda(\underline{z})$, then $(\underline{q}, \underline{z})$ is an equilibrium because $\underline{z}$ is on the low or high domain, which is locally stable according to Lemma 2. The remaining proof of Proposition 1 is straightforward from the definition of a symmetric local Nash equilibrium.

$\square$

Proposition 2. Suppose that q is the exposure to risky loans at equilibrium. If $g(q, n) > h(z^2)$, then there exists a unique low-risk equilibrium. If $g(q, n) = h(z^2)$, then there exist two equilibria, one of which is high-risk and the other low-risk. If $h(z^1) < g(q, n) < h(z^2)$, then there exist at least two equilibria, one of which is high-risk and the other low-risk, and there may exist a middle-risk equilibrium besides them. If $g(q, n) = h(z^1)$, then there exist two equilibria, one of which is high-risk and the other low-risk. If $g(q, n) < h(z^1)$, then there exists a unique high-risk equilibrium.

Proof. Use Definition 2 and Lemma 2.

$\square$

Proposition 3. A symmetric local Nash equilibrium is optimally credit-controlling if $\mu \geq 0$ or $\mu \leq -\lambda(\bar{z})$. It is risk-shifting if $-\lambda(\underline{z}) \leq \mu < 0$. It can be both risk-shifting and optimally credit-controlling if $-\lambda(\bar{z}) < \mu < -\lambda(\underline{z})$. There is no underinvesting equilibrium.

Proof. Let z satisfy (6). If $\mu \geq 0$, then $\mu + \lambda(z) \geq 0, \forall z$ so $q = \bar{q}$ by Lemma 3. From Definition 3, it is not risk-shifting.

If $\mu \leq -\lambda(\bar{z})$, then $\mu + \lambda(z) \leq 0$ for all z such that $g(q, n) = h(z)$, where $q \in [\underline{q}, \bar{q}]$. Therefore, $q = \underline{q}$ by Lemma 3. From Definition 3, it is also not risk-shifting.

If $-\lambda(\underline{z}) \leq \mu < 0$, then $\mu + \lambda(z) \geq 0, \forall z$ so $q \neq \underline{q}$. Since $\mu + \lambda(z) > \mu + \lambda(\underline{z}) \geq 0$ for all z such that $g(\bar{q}, n) = h(z)$, $q = \bar{q}$. From Definition 3, it is risk-shifting.

For the remaining case, $q = \bar{q}$ if $z = \bar{z}$ and $q = \underline{q}$ if $z = \underline{z}$. As neither $\underline{z}$ nor $\bar{z}$ is on the middle domain, they are both locally stable. Therefore, the banking sector attains multiple equilibria, one of which is optimally credit-controlling and the other is risk-shifting.

$\square$

There is no underinvesting equilibrium because the incentive contract eliminates the potential for an underinvesting equilibrium. Our results, however, suggest that there are often multiple equilibria one of which involves a high default risk or risk-shifting. Figure 2 and Table 1 illustrate the cases in which multiple equilibria can arise. On the one hand, the depositors may expect a low default risk and allow banks to offer low deposit rates, which increases the going concern value of banks. This lowers reservation rates. Moreover, this discourages risk-shifting because the gain from risk-shifting is proportional to the payoff of equity holders that is *conditional on bank survival*, which is amplified by the reservation rates of banks. On the other hand, the depositors may expect a high default risk and may require banks to offer high deposit rates, which decreases the continuation value of banks. Then, the opposite feedback occurs. Consequently, the banking sector can take excessive risk without regulation.

3.9 Implication for direct asset restrictions and antitrust policy

Next, we attempt to find the domain of policy parameters that induces banking robustness to the wide range of mean returns on risky loans.

Proposition 4. Let the exposure to risky loans at equilibrium be q. If $g(q,n) > h(z^2)$, then there exists a unique low-risk equilibrium. If $g(q,n) \geq h(z^1)$, then there exists a low-risk equilibrium. Moreover, $\frac{\partial g(q,n)}{\partial q} < 0$ and $\frac{\partial g(q,n)}{\partial n} < 0$.

Proof. Use Proposition 2. The last statement is obvious from the definition of $g(q,n)$. □

Proposition 5. The probability of certainly attaining a risk-shifting equilibrium is increasing in n and $\underline{q}$. The probability of certainly attaining an equilibrium that attains optimal credit control is decreasing in n and $\bar{q}$.

Proof. From Proposition 3, the probability of attaining only risk-shifting equilibria is $P[\mu \geq 0] + P[\mu \leq -\lambda(\bar{z})]$.

In addition, the probability of attaining only credit-enhancing equilibria is $P[-\lambda(\underline{z}) \leq \mu < 0]$.

Then our proof completes by: $\frac{\partial \bar{z}}{\partial n} > 0$, $\frac{\partial \underline{z}}{\partial n} > 0$, $\frac{\partial \bar{z}}{\partial \bar{q}} > 0$, and $\frac{\partial \underline{z}}{\partial \underline{q}} > 0$. □

Our findings suggest that direct asset restrictions or market concentration can eliminate the potential for a high-risk equilibrium by raising the Sharpe ratio of the bank equity. Moreover, these policies can improve credit control in the financial sector by expanding the domain of the mean returns on risky loans in which risk-shifting never occurs.

3.10 Welfare criteria for financial regulation

Although direct asset restrictions or market concentration can improve the soundness of the banking sector, these policies may harm the welfare of the depositors. By controlling the default risk, the depositors are better off with a higher exposure to risky loans because they can receive a higher interest. Moreover, market competition increases the surplus of the depositors by providing the depositors with more options for banks while decreasing the market power of banks. For verifying this claim, we characterize the ex-ante surplus of the depositors via a contingent valuation estimate of multinomial logit model [Petrin, 2002]. The value of the depositors by percentage is

$$D(\{\sigma_k\}_{k=1}^n) = \frac{M^I \ln\left\{\sum_{k=1}^n \exp(\alpha i_k^I)\right\} + (1 - M^I) \ln\left\{\sum_{k=1}^n \exp(\alpha i_k^N - \gamma\Phi(z_k)\right\} + \kappa}{\alpha}, \tag{10}$$

where κ is Euler constant (0.5772).

At a symmetric equilibrium, with deposit rates $i^I(\boldsymbol{\sigma})$ and $i^N(\boldsymbol{\sigma})$,

$$D(\boldsymbol{\sigma}) = \frac{\ln(n) + \alpha(M^I i^I(\boldsymbol{\sigma}) + (1 - M^I) i^N(\boldsymbol{\sigma}) - (1 - M^I)\gamma\Phi(z) + \kappa}{\alpha}.$$

At symmetric equilibrium, (1) and (2) suggest:

$$i^I(\boldsymbol{\sigma}) = i^N(\boldsymbol{\sigma}) - \xi\Phi(z)$$

$$i^N(\boldsymbol{\sigma}) = q(\mu + \nu\lambda(z)) - \frac{n}{\alpha(n-1)}.$$

Then, we obtain the value of the depositors at a symmetric equilibrium as follows:

$$D(\boldsymbol{\sigma}) = q(\mu + \nu\lambda(z)) - \xi\Phi(z) + \frac{\ln(n) + \kappa}{\alpha} - \frac{n}{\alpha(n-1)}. \tag{11}$$

The first term represents the gain from credit supply. We find that the shareholders' extra gain due to limited liability is partly transferred to the depositors because market competition forces the shareholders to give it to the depositors for collecting more funds. The second term represents the expected loss associated with a default. The last two terms are the gain from market competition.

Proposition 6. Suppose σ is an equilibrium strategy. The welfare of the depositors is strictly increasing in n. Moreover, it is increasing in $\bar{q}$ and decreasing in $\underline{q}$.

Proof. If $\mu + \nu\lambda(z) > 0$, then $q = \bar{q}$. As the welfare of the depositors is strictly increasing in q if $\mu + \nu\lambda(z) > 0$, it is increasing in $\bar{q}$.

If $\mu + \nu\lambda(z) = 0$, then there is no equilibrium.

If $\mu + \nu\lambda(z) < 0$, then $q = \underline{q}$. As the welfare of the depositors is strictly decreasing in q if $\mu + \nu\lambda(z) < 0$, it is decreasing in $\underline{q}$.

Finally, it is easy to check that the last two terms are strictly increasing in n.

□

As the welfare of the depositors is proportional to the gain from limited liability ($\nu\lambda(z)$), the depositors partly extract the gain of risk-shifting from the surplus of equity holders. This suggests the welfare of the depositors should be distinct from social welfare when risk-shifting takes place. When $\mu > 0$, both the depositors and the society are worse off with a lower $\bar{q}$. When $\mu < 0$, the depositors are worse off with a lower $\bar{q}$, whereas the society is better off with this. On the other hand, both the depositors and society are better off with a lower $\underline{q}$. From this insight, we establish the welfare criteria for financial regulation as follows.

Definition 4. The proposed policy harms social welfare if it either decreases n, decreases $\bar{q}$ when $\mu > 0$, or increases $\underline{q}$.

Thus, both direct asset restrictions and market concentration harm social welfare. In other words, there is a trade off between a sound banking system and social welfare. For the next two sections, we seek to make the banking sector attain both financial stability and optimal credit control without sacrificing social welfare.

4 Public Disclosure of Banks' Risk

Previous studies, such as Cordella and Yeyati [1998], suggest that public disclosure of banks' risk improves the soundness of the banking sector. Following these studies, we revisit the above game under the condition of a greater transparency of banks' risk. We find that this condition mitigates restrictions on bank assets and entries to the deposit market that regulators require to prevent a high default risk or risk-shifting. This is because banks internalize the negative impact of the tail risk on the amount they can collect from the uninsured depositors if they know that the depositors know their risks when choosing their banks. For implementing this policy, we predict the need for governmental intervention if equity holders cannot commit to injecting capital beyond the going concern value of banks. Although public disclosure of banks' risk unambiguously improves the soundness of the banking sector, its favorable effect decreases in the fraction of the insured depositors. This is because the insured depositors only consider deposit rates and are insensitive to the default risk of banks. This suggests that an increase in the coverage of deposit insurance ruins the favorable effect of transparent banking.

4.1 Modified game structure

Formally, step 4 of the previous game is altered to the following.

- The depositors observe both deposit rates and strategies of all the banks and choose their banks. Each bank k acquires the deposits from the same pool of insured and uninsured depositors.

Then, the uninsured depositors are excluded from this game because their belief regarding the default risk of each bank is internalized by the bank manager. We denote the modified game by Γ_1.

Definition 5. A strategy $\{\{\sigma_k\}_{k=1}^n, \boldsymbol{\sigma^I}\} \in \Sigma^n \times \Sigma^n$ is a symmetric local Nash equilibrium of Γ_1 if $\sigma_k = \sigma, \forall k = 1, ..., n$, and either
(i) there exist open rectangles $W \subset \Sigma, \forall k = 1, ..., n$, such that
$\sigma \in W$, $V(\sigma, \{\{\sigma\}_{k' \neq k}, \{\sigma, \{\sigma\}_{k' \neq k}\}, \boldsymbol{\sigma^I}\}) \geq V(\sigma', \{\{\sigma\}_{k' \neq k}, \{\sigma', \{\sigma\}_{k' \neq k}), \boldsymbol{\sigma^I}\})$, $\forall \sigma' \in W \setminus \sigma$, $\boldsymbol{\sigma^I} = \boldsymbol{\sigma}$, and $q \in (\underline{q}, \bar{q})$, or
(ii) there exist half-open rectangles, i.e. the cartesian products of right (left) half-open interval and open interval, $W \subset \Sigma$, $\forall k = 1, ..., n$, such that
$\sigma \in W$, $V(\sigma, \{\{\sigma\}_{k' \neq k}, \{\sigma, \{\sigma\}_{k' \neq k}\}, \boldsymbol{\sigma^I}\}) \geq V(\sigma', \{\{\sigma\}_{k' \neq k}, \{\sigma', \{\sigma\}_{k' \neq k}\}, \boldsymbol{\sigma^I}\})$, $\forall \sigma' \in W \setminus \sigma$, and $\boldsymbol{\sigma^I} = \boldsymbol{\sigma}$, and $q = \underline{q}(\bar{q})$.

4.2 Default decision

Under the modified game, the marginal value of the tail risk is changed to

$$\frac{\partial V(\sigma_k, \sigma_{-k})}{\partial z_k} = -\frac{\left(M^I s^I(\sigma_k, \sigma_{-k}) + (1 - M^I) s^N(\sigma_k, \sigma_{-k})\right)}{r + \Phi(z_k)} \times \left\{ \begin{array}{c} \phi(z_k)(1 - \Phi(z_k))(1 - \theta^I(\sigma_k, \sigma_{-k}))\xi \\ +\phi(z_k)\left(\frac{1+r}{r+\Phi(z)}\pi(\sigma_k, \sigma_{-k}) - q_k \nu(\lambda(z_k) - z_k)\right) \end{array} \right\}. \tag{12}$$

At symmetric equilibrium, the first-order condition is modified to

$$\frac{1+r}{r+\Phi(z)} \frac{n}{\alpha(n-1)} - q\nu\left[\lambda(z) - z\right] = -(1 - \Phi(z))(1 - M^I)\xi < 0. \tag{13}$$

Rewriting the equation that determines the reservation rate yields

$$\frac{n}{\alpha(n-1)q\nu} + \frac{(r + \Phi(z))(1 - \Phi(z))(1 - M^I)\xi}{(1+r)q\nu} = \frac{r + \Phi(z)}{1+r}\left[\lambda(z) - z\right]. \tag{14}$$

Compared with the original equation, the LHS unambiguously shifts up. We define the new term of the LHS as the cost of losing the uninsured deposits $c^N(q, z, M^I)$. $c^N(q, z, M^I)$ strictly increases in z for $z \leq \frac{1-r}{2}$ and strictly decreases in z for $z \geq \frac{1-r}{2}$. Moreover, $c^N(q, z, M^I)$ is strictly decreasing in M^I. Furthermore, $\lim_{z \to \infty} c^N(q, z, M^I) = 0$ and

$\lim_{z\to-\infty} c^N(q,z,M^I) = \frac{(1-M^I)\xi}{q\nu}$. Therefore, there exists at least one root satisfying (14) for any q.

Unlike the previous game, (13) implies that the going concern value of the bank has to be less than the shortfall needed for the bank to continue business when the reservation rate is realized. This implies that equity holders have an *ex-ante* incentive to lower the reservation rate even if it is unprofitable *ex-post*, because they can collect more money from the uninsured by promising that they would inject capital beyond the continuation value. Accordingly, conditions required for local stability are modified to

$$\frac{\lambda'(z)}{n} - h'(z) \geq \phi(z)\frac{\xi(1-M^I)}{(1+r)q\nu}\left[\alpha\xi M^I(r+\Phi(z))(1-\Phi(z))\frac{(n-1)^2}{n^2} - (1-r-2\Phi(z))\right] \tag{15}$$

$$\frac{\lambda'(z)}{n} - h'(z) > \phi(z)\frac{\xi(1-M^I)}{(1+r)q\nu}\left[\alpha\xi M^I(r+\Phi(z))(1-\Phi(z))\frac{(n-1)^2}{n^2} - (1-r-2\Phi(z))\right]. \tag{16}$$

Then the stability condition becomes more restrictive than before. Even if z is not in the middle domain, it may not satisfy (15).

Lemma 4. If M^I is sufficiently large, then z on the low and high domains satisfies (16). If M^I is sufficiently small, then z on the low domain satisfies (16).

Proof. It is obvious from (16). For the second statement, the RHS of (16) becomes negative with sufficiently small M^I because $1-r-2\Phi(z) > 0, \forall z \leq z^1$ for modestly low r.

□

4.3 Results

The choice of the overall risk is determined in the same way as before. We can characterize the equilibrium of the modified game as follows.

Proposition 7. If σ is a symmetric local Nash equilibrium strategy of Γ_1, σ satisfies (14), (15), and either $q = \bar{q} \wedge \mu > -\nu\lambda(z)$ or $q = \underline{q} \wedge \mu < -\nu\lambda(z)$. Conversely, if σ satisfies (14), (16), and either $q = \bar{q} \wedge \mu > -\nu\lambda(z)$ or $q = \underline{q} \wedge \mu < -\nu\lambda(z)$, σ is a symmetric local Nash equilibrium strategy of Γ_1.

Proof. Similar to the proof of Proposition 1.

□

Proposition 8. Suppose M^I is sufficiently small or large. If there exists a unique low-risk equilibrium under Γ, then there exists a unique low-risk equilibrium under Γ_1 as well.

However, the converse is not true. Moreover, if there exists a unique low-risk equilibrium under Γ_1, then there remains a unique low-risk equilibrium under Γ_1 when M^I decreases. However, the converse is not true.

Proof. When M^I is sufficiently small or large, Lemma 4 suggests that the stability condition on the low domain of z is automatically satisfied. Let the exposure to risky loans at equilibrium be q. If $g(q,n) + c^N(q,z,M^I) > h(z^2), \forall z \geq z^2$, then there exists a unique low-risk equilibrium. If $g(q,n) > h(z^2)$, then $g(q,n) + c^N(q,z,M^I) > h(z^2), \forall z \geq z^2$. This proves the first part.

Moreover, $\frac{\partial c^N(q,z,M^I)}{\partial M^I} < 0$. This proves the second part. For the converse, you can see our calibration results described later. There you find that public disclosure of banks' risk makes the banking sector attain a unique low-risk equilibrium, whereas the banking sector can attain a high-risk equilibrium without disclosure. Moreover, you find the case in which the larger fraction of the insured depositors mitigates the favorable effect of transparent banking that eliminates the potential for a high-risk equilibrium.

□

Proposition 9. Suppose M^I is sufficiently small and $g(\bar{q},n) \geq h(z^1)$ or M^I is sufficiently large. The probability of certainly attaining a risk-shifting equilibrium is greater under Γ than that under Γ_1. On the other hand, the probability of certainly attaining an equilibrium that achieves optimal credit control is greater under Γ_1 than that under Γ. Moreover, under Γ_1, the probability of certainly attaining a risk-shifting equilibrium is increasing in M^I, while that of certainly attaining a equilibrium that achieves optimal credit control is decreasing in M^I.

Proof. When M^I is sufficiently large, there exists a either low-risk or high-risk equilibrium under Γ_1, regardless of q. When M^I is sufficiently small and $g(\bar{q},n) \geq h(z^1)$, there exists at least a low-risk equilibrium under Γ_1, regardless of q.

Define $\underline{z_1} = \inf\{z | g(\underline{q},n) + c^N(\underline{q},z,M^I) = h(z)\}$ and $\bar{z_1} = \sup\{z | g(\bar{q},n) + c^N(\bar{q},z,M^I) = h(z)\}$. It is easy to show $\underline{z_1} < \bar{z_1}$.

If $\mu \geq 0 \vee \mu \leq -\nu\lambda(\bar{z_1})$, then the system certainly attains equilibria with optimal credit control.

If $-\nu\lambda(\underline{z_1}) \leq \mu < 0$, then the system certainly attains risk-shifting equilibria. $c^N(q,z,M^I) > 0$ implies $\bar{z} > \bar{z_1}$ and $\underline{z} > \underline{z_1}$. This proves the first statement of our claim.

$\frac{\partial c^N(q,z,M^I)}{\partial M^I} < 0$ implies $\frac{\partial \bar{z_1}}{\partial M^I} > 0$ and $\frac{\partial \underline{z_1}}{\partial M^I} > 0$. This proves the second statement of our claim.

□

Our results suggest that there is a chance of strictly improving the soundness of financial sector by disclosing banks' risk to households. Conditional on the existence of an equilibrium, the public disclosure of banks' risk decreases the default risk and induces the system to attain optimal credit control. Consequently, regulators can relax direct asset restrictions and enhance market competition without sacrificing the soundness of the banking sector. However, the favorable effect of transparent banking decreases the fraction of the insured depositors because they do not have an incentive to monitor the default risk of each bank. Figure 3 illustrates how the transparent banking can improve the soundness of the banking sector and how its favorable effect is offset by a greater coverage of deposit insurance.

4.4 Implication for the role of government during financial crises

Public disclosure of banks' risk can improve the soundness of banking sector, because it reminds banks of the monitoring and threatening roles of the depositors, which is essential for banking prudence, according to Diamond and Rajan [2001]. However, it is not clear if shareholders can commit to injecting capital beyond the promised amount. Once the funds are collected from the depositors, they may not have an incentive to raise more capital than the going concern value of the bank. In this case, governments may need to inject capital on behalf of the incumbent shareholders. If the depositors know that governments inject capital on behalf of the shareholders for the amount that cannot be committed, then they can believe that the default risk is lower than the *ex-post* optimal rate for the incumbent shareholders.

Unlike too-important-to-fail (TITF) subsidies, the government bailout is unrelated to the bank size. Therefore, shareholders are unwilling to take excessive exposure to risky loans, raise deposit rates, and increase market share. In this sense, the government intervention does not cause a moral hazard.

On the other hand, deposit insurance is not compatible with transparent banking. Governments often increase the coverage for deposit insurance to prevent bank runs in an economic downturn; however, this ruins the favorable effect of disclosing banks' risk. Therefore, if regulators try to achieve financial stability by the public disclosure of banks' risk, they should limit the coverage of deposit insurance to the minimum level required to prevent bank runs.

Thus, if governments require banks to disclose their strategies to households, they should help inject capital to the level maximizing the *ex-ante* value of equity while keeping the coverage of deposit insurance to the minimum level required to prevent bank runs.

5 Debt-type Managerial Compensation

In the previous sections, managerial interests are perfectly aligned with the incentives of shareholders. In this section, we link managerial compensation to the tail risk of a bank. This induces a conflict of interest between a manager and the shareholders. We show that the agency problem between them rather helps a bank achieve optimal credit control without sacrificing social welfare; moreover, our proposal is unaffected by the fraction of the insured depositors. Furthermore, the prevention of risk-shifting can also eliminate the potential for a high-risk equilibrium when $\mu < 0$, if regulators cap the minimum exposure to risky loans. While regulators are reluctant to restrict the *maximum* exposure to risky loans, they are justified to restrict the *minimum* exposure to risky loans.

5.1 Modified game structure

Even if the manager is able to choose the exposure to risky loans independently, it is difficult for them to mandate the shareholders to inject the specified amount of capital. Therefore, we assume that the manager only determines the exposure to risky loans while the shareholders determine the tail risk. We change steps 1, 3, and 5 of Γ as follows.

- At each bank k, the incumbent equity holders hire a manager under an incentive contract as they do in Γ. However, the government imposes taxes on the compensation of the manager, which are linked to the credit default swap (CDS) spread of the bank. Therefore, the manager's compensation is:
  $W(q_k, z_k, \sigma_{-k}) = W_0 + \delta^E V(q_k, z_k, \sigma_{-k}) - \tau(C(z_k))$, where W_0 is a fixed wage, δ^E is the shares of equity ($\delta^E > 0$), and $\tau(C(z_k))$ is the tax associated with the CDS spread of the bank. We also denote it by $f(z_k) = \tau(C(z_k))$. The manager of the bank immediately receives W_0.

- The manager of each bank simultaneously chooses q_k and the shareholders of each bank determines z_k after observing q_k. After observing both, the manager determines deposit rates and announces them to households.

- $V(q_k, z_k, \sigma_{-k})$ and $C(z_k)$ are determined by efficient markets. Each bank pays bonuses to the manager following the incentive contract and the government collects taxes $\tau(C(z_k))$ from the manager.

We denote the modified game by Γ_2.

Definition 6. A strategy $(\{q_k\}_{k=1}^n, \{z_k\}_{k=1}^n, \boldsymbol{\sigma^N}, \boldsymbol{\sigma^I}) \in Q^n \times \mathbb{R}^n \times \Sigma^n \times \Sigma^n$ is a symmetric local Nash equilibrium of Γ_2 if $q_k = q, z_k = z, \forall k = 1, ..., n$, and either
(i) there exist open intervals $M \subset Q$,$\forall k = 1, ..., n$, such that
$q \in M$, $W(q, z, \{\sigma\}_{k' \neq k}, \boldsymbol{\sigma^N}, \boldsymbol{\sigma^I}) \geq W(q', z, \{\sigma\}_{k' \neq k}, \boldsymbol{\sigma^N}, \boldsymbol{\sigma^I})$, $\forall q' \in M \setminus q$, $\boldsymbol{\sigma^N} = \boldsymbol{\sigma^I} = \boldsymbol{\sigma}$, and $q \in (\underline{q}, \bar{q})$, or
(ii) there exist right (left) half-open intervals, $M \subset Q$, $\forall k = 1, ..., n$, such that
$q \in M$, $V(q, z, \{\sigma\}_{k' \neq k}, \boldsymbol{\sigma^N}, \boldsymbol{\sigma^I}) \geq V(q', z, \{\sigma\}_{k' \neq k}, \boldsymbol{\sigma^N}, \boldsymbol{\sigma^I})$, $\forall q' \in M \setminus q$, $\boldsymbol{\sigma^N} = \boldsymbol{\sigma^I} = \boldsymbol{\sigma}$, and $q = \underline{q}(\bar{q})$, and
there exist open intervals $E \subset \mathbb{R}$,$\forall k = 1, ..., n$, such that
$z \in E$, $V(q, z, \{\sigma\}_{k' \neq k}, \boldsymbol{\sigma^N}, \boldsymbol{\sigma^I}) \geq V(q, z', \{\sigma\}_{k' \neq k}, \boldsymbol{\sigma^N}, \boldsymbol{\sigma^I})$, $\forall z' \in E \setminus z$, $\boldsymbol{\sigma^N} = \boldsymbol{\sigma^I} = \boldsymbol{\sigma}$.

5.2 Portfolio choice

The optimality condition for the tail risk is the same as the one in Γ, but the marginal value of the overall risk is modified to

$$\begin{aligned}&\frac{\partial V(\sigma_k,\sigma_{-k})}{\partial q_k}-\frac{f'(z_k)}{\delta^E}\frac{dz_k}{dq_k}\\ =&\frac{1-\Phi(z_k)}{r+\Phi(z_k)}\left(M^I s^I(\sigma_k,\sigma_{-k})+(1-M^I)s^N(\sigma_k,\sigma_{-k})\right)(\mu+\nu\lambda(z_k))-\frac{f'(z_k)}{\delta^E}\frac{dz_k}{dq_k}.\end{aligned} \tag{17}$$

Unlike the previous games, the manager takes into account the marginal cost of increasing the default risk determined by the shareholders because of the additional exposure to risky loans. Even if the manager is not able to choose the tail risk, he can affect it by adjusting the exposure to risky loans. At symmetric equilibrium, the marginal value of the overall risk is as follows:

$$\frac{1-\Phi(z)}{r+\Phi(z)}\frac{\mu+\nu\lambda(z)}{n}-\frac{f'(z)}{\delta^E}\frac{\frac{n}{\alpha(n-1)}-\frac{q[\mu+\nu\lambda(z)]}{n}}{q^2\nu\left(\frac{\lambda'(z)}{n}-h'(z)\right)}. \tag{18}$$

5.3 Tax on the tail risk of a bank

Our goal is to eliminate risk-shifting without sacrificing social welfare. If (18) is negative for all q if $\mu<0$ and positive for all q if $\mu>0$, then $\bar{q}$ becomes an equilibrium exposure to risky loans whenever $\mu>0$ while $\underline{q}$ becomes an equilibrium exposure to risky loans whenever $\mu<0$. If $f'(z)\geq 0$, we can induce the equilibrium by setting $f'(z)$ as the following:

$$f'(z)=\delta^E\frac{g^{-1}(z,n)^2\nu\left(\frac{\lambda'(z)}{n}-h'(z)\right)}{\frac{n}{\alpha(n-1)}-\frac{g^{-1}(z,n)\nu\lambda(z)}{n}}\frac{1-\Phi(z)}{r+\Phi(z)}\frac{\nu\lambda(z)}{n}. \tag{19}$$

Here $g^{-1}(z,n)$ satisfies $g(g^{-1}(z,n),n)=h(z)$, which is well-defined and continuous. Then, we find that the sufficient condition for $f'(z)\geq 0$ is summarized by the following Lemma.

Lemma 5. If $\frac{n}{\alpha(n-1)}-\frac{g^{-1}(\bar{z},n)\nu\lambda(\bar{z})}{n}>0$, then $f'(z)\geq 0$, where z is an equilibrium tail risk. If n is sufficiently large, then $\frac{n}{\alpha(n-1)}-\frac{g^{-1}(\bar{z},n)\nu\lambda(\bar{z})}{n}>0$.

Proof. As $\frac{\lambda'(z)}{n}-h'(z)\geq 0$ at equilibrium, $\frac{n}{\alpha(n-1)}-\frac{g^{-1}(z,n)\nu\lambda(z)}{n}>0$ is sufficient to achieve $f'(z)\geq 0$, where z is an equilibrium tail risk.
As $z\leq\bar{z}$, $\frac{n}{\alpha(n-1)}-\frac{g^{-1}(\bar{z},n)\nu\lambda(\bar{z})}{n}>0\Rightarrow\frac{n}{\alpha(n-1)}-\frac{g^{-1}(z,n)\nu\lambda(z)}{n}>0$.

The second statement is confirmed by $\lim_{n\to\infty} \frac{n}{\alpha(n-1)} - \frac{g^{-1}(\bar{z},n)\nu\lambda(\bar{z})}{n} = \frac{1}{\alpha} > 0$.

$\square$

Note that market competition is a complement to the avoidance of risk-shifting, whereas it was a substitute to the prevention of risk-shifting in our previous results. In fact, the sufficient condition for the previous Lemma is satisfied if n is sufficiently large. On one hand, an increase in the exposure to risky loans decreases the Sharpe ratio of the bank equity by increasing the portfolio return volatility. On the other hand, it increases the continuation value of the bank as it enables the bank to collect more funds from the depositors by raising default rates when $\mu + \lambda(z) > 0$. In competitive markets, the former effect dominates the latter effect because the increase in market share associated with the increase in deposit rates is smaller.

Suppose that $\frac{n}{\alpha(n-1)} - \frac{g^{-1}(\bar{z},n)\nu\lambda(\bar{z})}{n} > 0$. Setting the boundary condition $f(\underline{z}) = 0$, the optimal debt-based tax is characterized by the following:

$$f(z) = \int_{\underline{z}}^{z} 1[f'(x) \geq 0] f'(x) dx. \tag{20}$$

We can assign any value to the marginal increase in tax with respect to tail risk for z that cannot be at equilibrium; therefore, we set 0 for z that satisfies $\frac{\lambda'(z)}{n} - h'(z) < 0$ and hence $f'(z) < 0$. Our goal is to construct the tax as a function of the CDS spread.

A no arbitrary condition suggests:

$$C(z_k) = \xi\Phi(z_k).$$

Then, z can be expressed as the monotonic transformation of the CDS spread and $f(z)$ can be expressed as a function of the CDS spread. Finally, we characterize the optimal compensation as

$$W(q_k, z_k, \sigma_{-k}) = W_0 + \delta^E V(q_k, z_k, \sigma_{-k}) - f\left(\Phi^{-1}\left(\frac{C(z_k)}{\xi}\right)\right), \tag{21}$$

where $f(.)$ satisfies (20).

Proposition 10. Suppose that n satisfies $\frac{n}{\alpha(n-1)} - \frac{g^{-1}(\bar{z},n)\nu\lambda(\bar{z})}{n} > 0$. Every equilibrium is optimally credit-controlling under Γ_2 if the government assigns the tax: $\tau(C(z_k)) = f\left(\Phi^{-1}\left(\frac{C(z_k)}{\xi}\right)\right)$, where $f(.)$ satisfies (20).

Proposition 10 suggests that regulators can eliminate the potential for risk-shifting without direct asset restrictions or market concentration. Moreover, this approach is independent of depositor composition; therefore, it is neutral to the coverage of deposit insurance. Furthermore, this compensation structure can be implemented by the tax linked to the CDS spread of a bank.

In addition, note that the prevention of risk-shifting can eliminate the potential for a high-risk equilibrium in an economic downturn. If $\underline{q}$ is small enough to satisfy $g(\underline{q}, n) > h(z^2)$, then we can eliminate the potential for a high-risk equilibrium because the manager voluntarily reduces exposure to risky loans to the minimum level if $\mu < 0$.

Proposition 11. Suppose that n satisfies $\frac{n}{\alpha(n-1)} - \frac{g^{-1}(\bar{z},n)\nu\lambda(\bar{z})}{n} > 0$ and $\underline{q}$ satisfies $g(\underline{q}, n) > h(z^2)$. The banking sector attains a unique low-risk equilibrium when $\mu < 0$ under Γ_2 if the government assigns the tax $\tau(C(z_k)) = f\left(\Phi^{-1}\left(\frac{C(z_k)}{\xi}\right)\right)$, where $f(.)$ satisfies (20).

Enhanced market competition lowers the Sharpe ratio of the bank equity. Therefore, regulators may need to lower $\underline{q}$ to maintain the Sharpe ratio of the bank equity. In the previous findings, restrictions on the maximum exposure to risky loans are not fully recommended because they harm credit enhancement in the society. However, restrictions on the minimum exposure to risky loans do not harm social welfare. This is because the society is willing to lower exposure to the risky loans that are unprofitable. Although the depositors may be worse off by a lower $\underline{q}$, the decrease in their surplus is associated with a reduction on the gain from risk-shifting that the depositors extract from the surplus of shareholders. Consequently, regulators are justified to increase the Sharpe ratio of the bank equity by providing liquidity to banks and lowering $\underline{q}$ in an economic downturn, even though they may reduce the welfare of the depositors.

6 Calibration

In this section, we calibrate the model and design the policy packages that improve the soundness of the US commercial banking sector.

6.1 Data

We obtain the demand parameters from Egan et al. [2014]. As these authors reported the demand estimates separately for each type of depositors, we use the middle of them for our calibration. From the demand estimates for the uninsured depositors, we can recover the fire-sale discount rate by dividing the sensitivity to deposit rates with the sensitivity to the default risk. We find that the corresponding recovery rate is 50%, which is in line with previous studies, such as Carrizosa and Ryan [2013].

We use the data from the Federal Reserve H8 to obtain the exposure to risky loans. We compute cash as well as Treasury and agency securities as a proportion of total assets and subtract it from 1. As Egan et al. [2014] focused on large US banks, we use the data for large domestically chartered commercial banks. As regulators are interested in whether the current policy is robust enough, we estimate the maximum exposure to the risky loans by using the data as of November 12, 2014. Note that the expected return on risky loans is likely to be above the risk-free rate in 2014. Then, banks are likely to invest in risky loans as much as possible. Consequently, the exposure to risky loans that we estimated from the data as of November 12, 2014 can be considered as the *maximum* exposure to risky loans to date.

The standard deviation of the return on risky loans is the remaining input for the calibration. Egan et al. [2014] report the calibrated standard deviation of the return on deposits as of March 31, 2009. Since we can estimate the exposure to the risky loans by using the Federal Reserve data as of March 31, 2009. We can recover the standard deviation of the returns to risky loans in a consistent manner. The table given below summarizes the parameters that we use for calibration.

α	40
ξ	0.5
ν	0.2
r	0.05
$\bar{q}$	0.7

6.2 Results

We calculate the boundary of the maximum exposure to risky loans and the number of banks *below* which the banking sector attains a unique low-risk equilibrium. Moreover, we compute the threshold mean return on risky loans *above* which risk-shifting can occur. For each, we solve the threshold under the following conditions: (1) without disclosure of banks' risk and (2) with public disclosure of banks' risk where the fraction of the insured depositors is 10%, 50%, and 90%. Figure 4 summarizes the results for the former estimates and Table 2 shows the latter estimates.

We find that the potential for a high-risk or risk-shifting equilibrium is persistent without the disclosure of banks' risk. Even if there are only two banks in the sector, where banks take the least risk, there exists an equilibrium where the probability of default is almost 1 and banks can undergo the maximum exposure to risky assets even if risky loans underperform riskless bonds by more than 40 pp per year. Therefore, the impact of eliminating the potential for the catastrophic equilibrium is substantial.

Thus, if regulators attempt to keep the current level of credit enhancement, the public disclosure of banks' risk is necessary for preventing an unfavorable equilibrium. However, this can eliminate the potential for such equilibrium only if there are only two banks, if regulators insure half of the depositors[10]. Therefore, if regulators try to maintain the current maximum exposure to risky loans, they have to make the banking sector extremely concentrated even when they order the transparency of banks' risk. If regulators attempt to enhance market competition, they need to set a severe cap on the maximum exposure to risky loans. Figure 5 and Table 3 summarize the impact of capping the maximum exposure to risky loans at 50%. If half of the depositors have to be covered by deposit insurance,

[10] In the largest commercial banks, approximately half of deposits are uninsured [Egan et al., 2014].

then, according to our estimates, the banking sector can accommodate up to four banks. Therefore, regulators have to restrict maximum exposure to increase the number of banks in the sector. Consequently, regulators cannot avoid to harm social welfare to achieve a sound banking system.

Figures 6, 7, and 8 show the optimal tax schedule by the number of banks. We find that regulators cannot prevent the potential for a risk-shifting equilibrium that lies on the high domain of a tail risk if there are several banks in the sector. According to our estimates, the maximum tail risk that can emerge at equilibrium is five when there are thirty banks. Then, this satisfies the condition for assigning the tax on the full support of the tail risk. If the minimum exposure to risky loans is 28%, the banking sector can attain a low-risk equilibrium that also achieves optimal credit control in an economic downturn, even if there are thirty banks in the sector. The minimum exposure to risky loans can be reduced by liquidity provision to banks. Consequently, regulators can fully hedge against risk-shifting and eliminate the potential for a high-risk equilibrium in an economic downturn if they accommodate thirty banks, tax on the executive compensation based on the tail risk of banks, and allow banks to convert 60% of their risky assets into liquid assets during crises. This policy package sacrifices neither social welfare nor the coverage of deposit insurance.

7 Discussion

We find that direct asset restrictions or market concentration can reduce the potential for equilibria to exist in either high default risk or risk-shifting, although they harm social welfare. Disclosing the risks that banks pose to depositors can relax restrictions on the level of risky loans and market concentration required to prevent catastrophic consequences while its favorable impact is offset by deposit insurance. Debt-type managerial compensation can eliminate the potential for risk-shifting. Moreover, it eliminates the potential for high default risk in an economic downturn when combined with liquidity provision to banks during crises. Our findings are qualitatively consistent with existing studies.

Without regulation, the potential for a catastrophic equilibrium exists, where banks can choose to default with the probability of almost 1 and accept excessive exposure to the risky loans that underperform the riskless bonds by more than 40 pp per year. Therefore, the effects of the studied policies may have been underestimated in previous studies, which do not take into account the potential for such a catastrophic equilibrium. We are also surprised to find that debt-type managerial compensation is complementary to market competition and liquidity provision in an economic downturn, whereas it is neutral to deposit insurance. As a result, this policy can improve the soundness of the banking sector without harming social welfare or reducing the coverage of deposit insurance.

On the basis of our findings, we propose the two policy packages that improve the soundness of the banking sector. If regulators want to completely eliminate the potential for high default risk or risk-shifting, then they should order banks to disclose the risks they

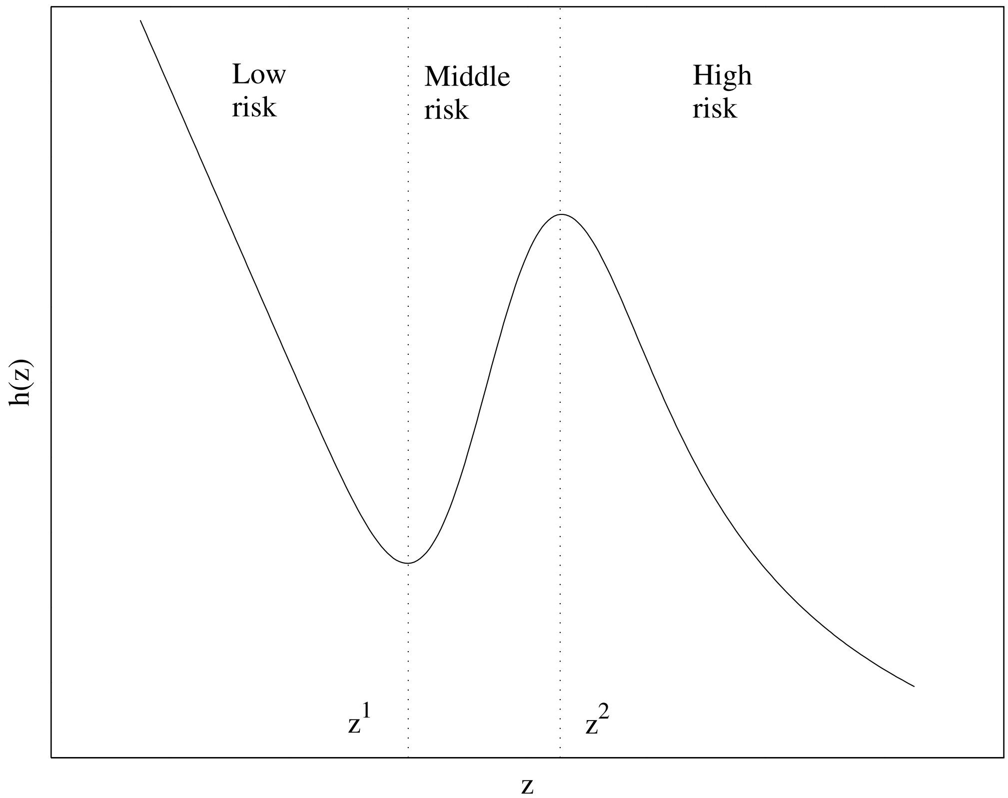
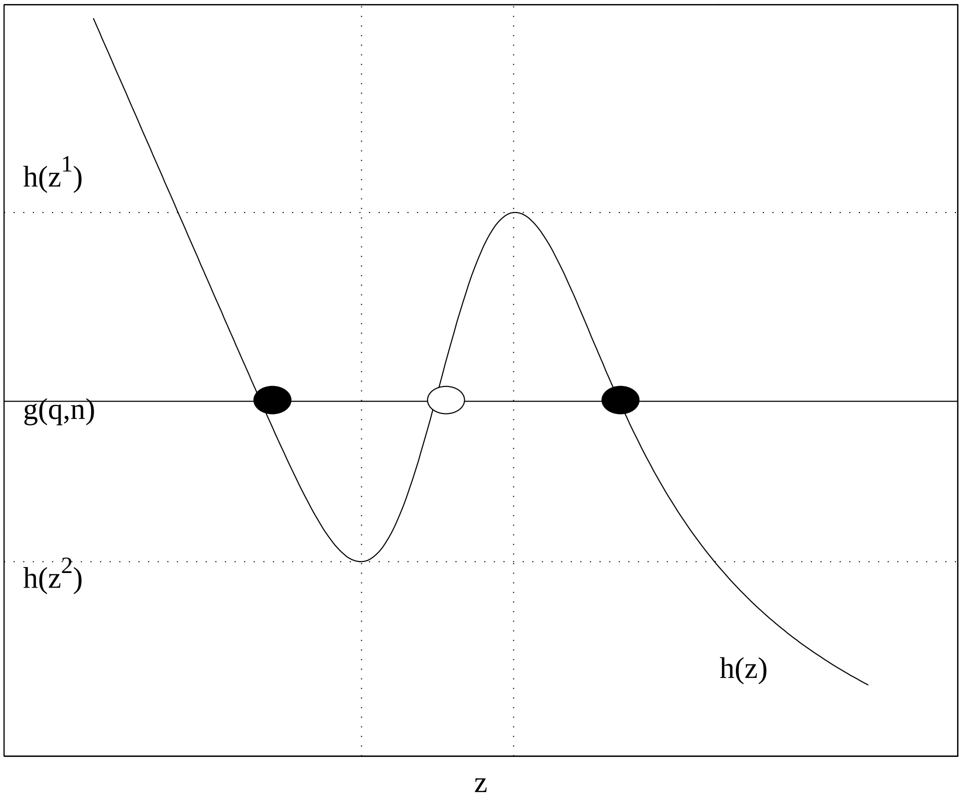


Figure 1: Cost of capital and the classification of tail risk

pose to depositors and set the coverage of deposit insurance to the minimum level required to prevent bank runs. Moreover, they need to restrict bank assets or entries to deposit market. If the maximum exposure to the risky loans is 70%, then they can accommodate only two banks. If it is 50%, then they can accommodate up to four banks. They cannot avoid harming social welfare, but the impact of fully eliminating an unfavorable equilibrium is substantial.

On the other hand, if regulators want to fully eliminate the potential for risk-shifting and high default risk only in an economic downturn, they can introduce debt-type managerial compensation without requiring public disclosure of banks' risk. At the same time, they need to accommodate thirty banks and provide sufficient liquidity to banks during crises. According to our estimates, they need to allow banks to convert 60% of their risky assets into safe assets in an economic downturn. The combination of these policies, however, does not harm social welfare. Moreover, it is neutral to the coverage of deposit insurance.



The ellipses represent potential equilibria. The shaded ellipses are locally stable. The unshaded ellipse can be locally unstable.

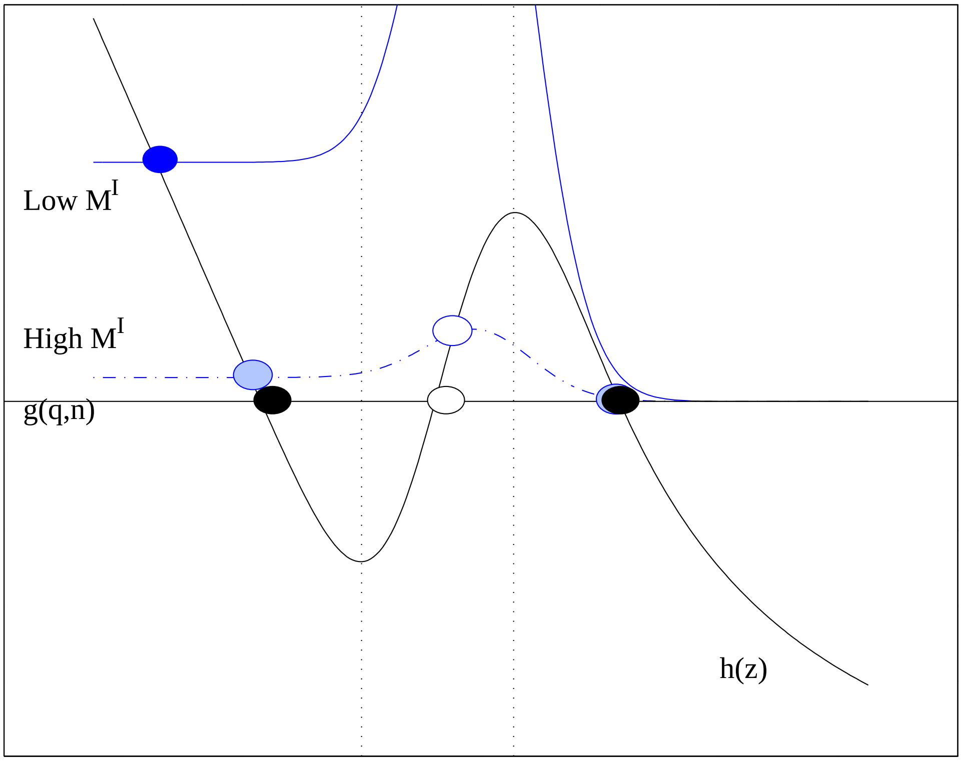
Figure 2: Potential for multiple equilibria (tail risk)

The mean return to the risky loans	$\mu \leq -\nu\lambda(\bar{z})$	$-\nu\lambda(\bar{z}) < \mu < -\nu\lambda(\underline{z})$	$-\nu\lambda(\underline{z}) \leq \mu$
The exposure to the risky loans at equilibrium	$\underline{q}$	$\underline{q}, \bar{q}$	$\bar{q}$

Table 1: Potential for multiple equilibria (overall risk)

Number of banks	**2**	**3**	**4**	**5**	**6**	**7**	**8**
No disclosure	-0.490	-0.698	-0.797	-0.855	-0.894	-0.921	-0.942
With disclosure ($M^I = 0.9$)	-0.475	-0.698	-0.797	-0.855	-0.894	-0.921	-0.942
With disclosure ($M^I = 0.5$)	-0.000	-0.695	-0.796	-0.855	-0.894	-0.921	-0.942
With disclosure ($M^I = 0.1$)	-0.000	-0.692	-0.796	-0.855	-0.894	-0.921	-0.942

Table 2: Minimum mean return to risky loans above which risk-shifting can occur



The ellipses represent potential equilibria. The shaded ellipses are locally stable. The unshaded ellipse can be locally unstable.

Figure 3: Effect of transparent banking by the fraction of insured depositors

Number of banks	**2**	**3**	**4**	**5**	**6**	**7**	**8**
With disclosure ($\bar{q} = 0.7$)	-0.000	-0.695	-0.796	-0.855	-0.894	-0.921	-0.942
With disclosure ($\bar{q} = 0.5$)	-0.000	-0.000	-0.000	-0.512	-0.587	-0.617	-0.636

We assume that the fraction of insured depositors is 0.5.

Table 3: Effect of direct asset restrictions on the minimum mean return to the risky loans above which risk-shifting can occur

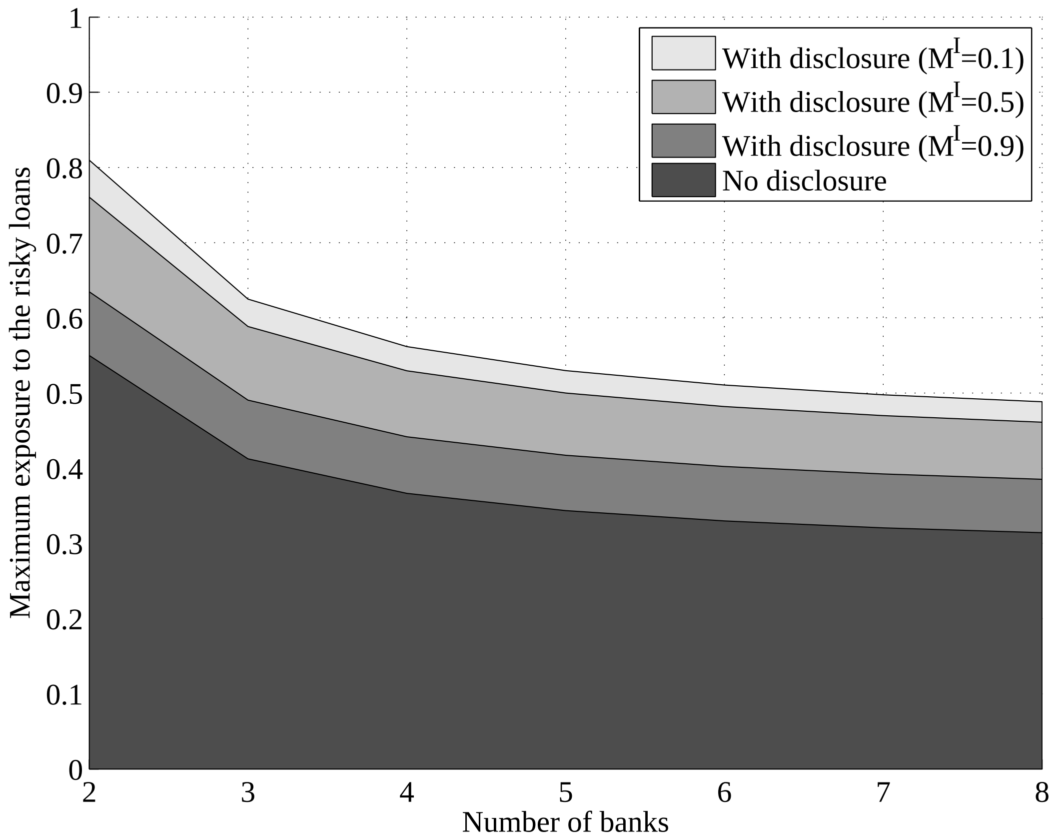
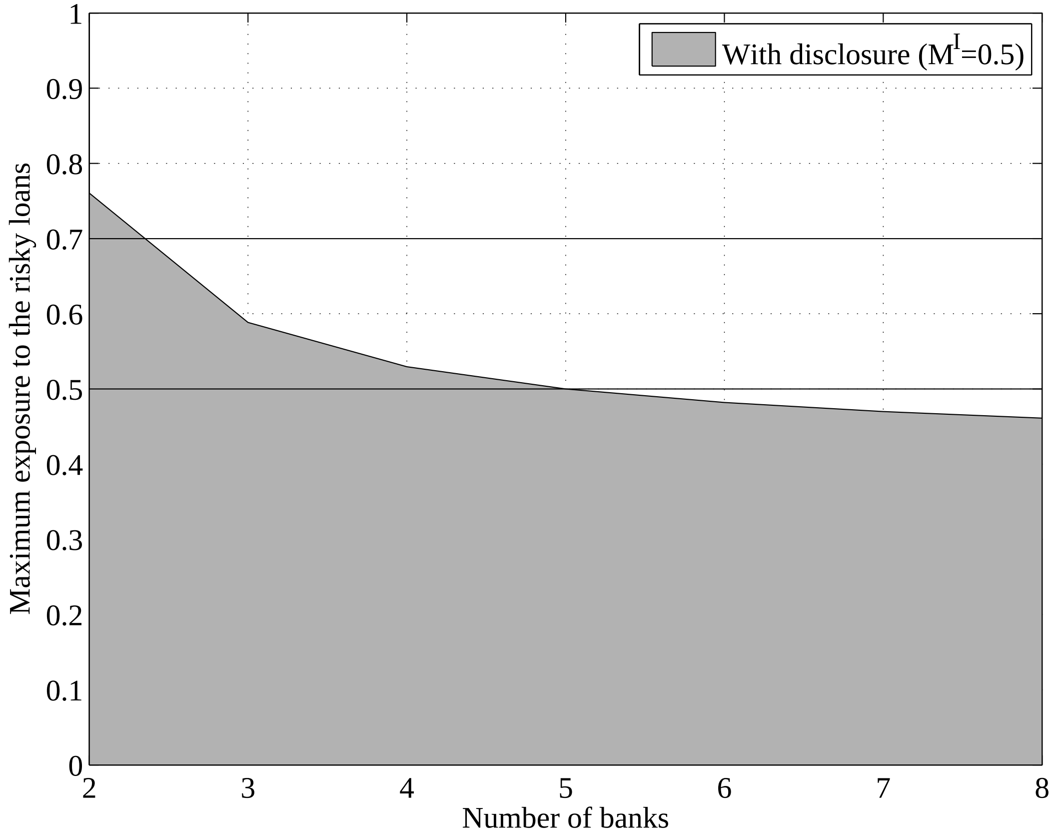


Figure 4: Boundary of policy parameters below which the financial system attains a unique low-risk equilibrium



The connected lines represent the maximum exposure to the risky loans of 0.7 and 0.5.

Figure 5: Effect of direct asset restrictions on the maximum number of banks below which the banking sector attains a unique low-risk equilibrium

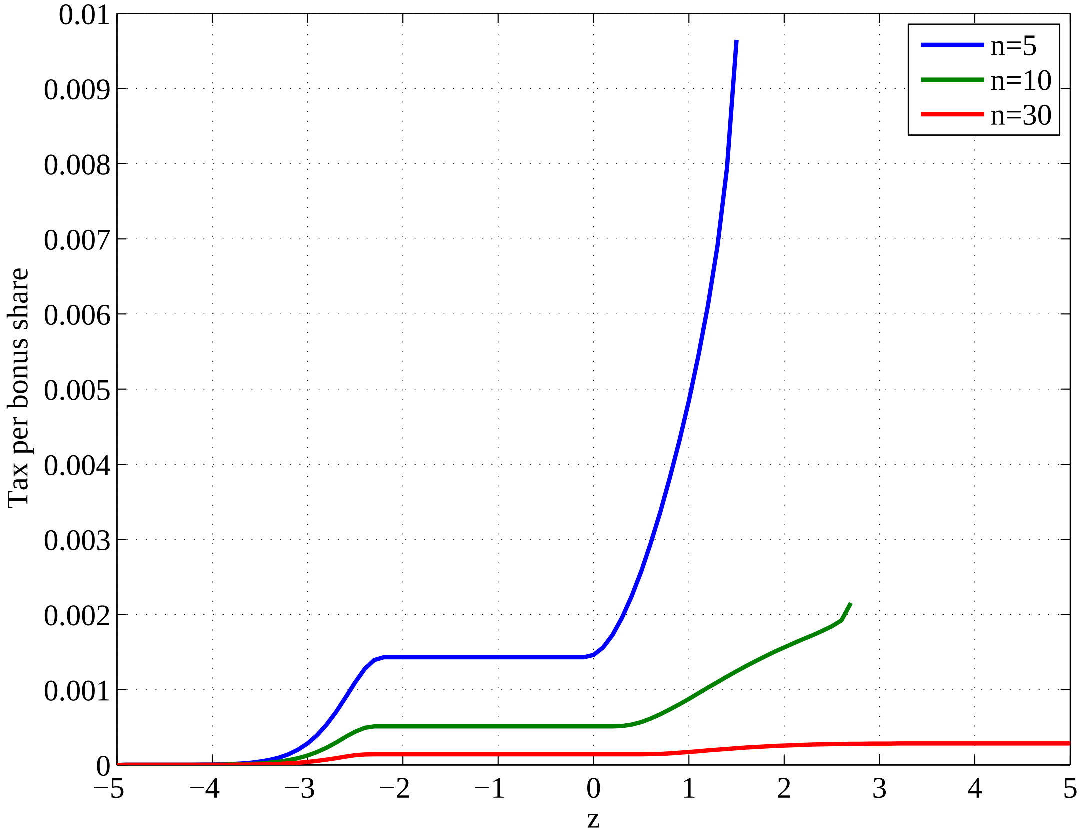


Figure 6: Optimal tax schedule as a function of tail risk

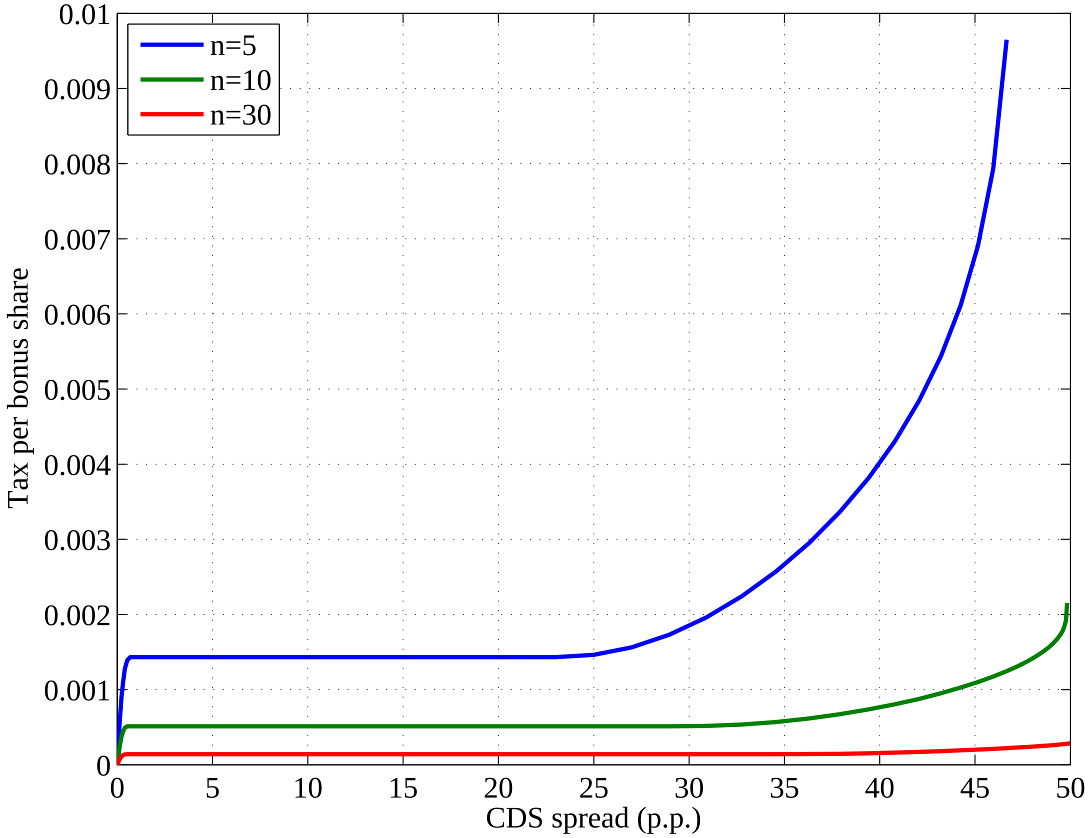


Figure 7: Optimal tax schedule as a function of CDS spread (I)

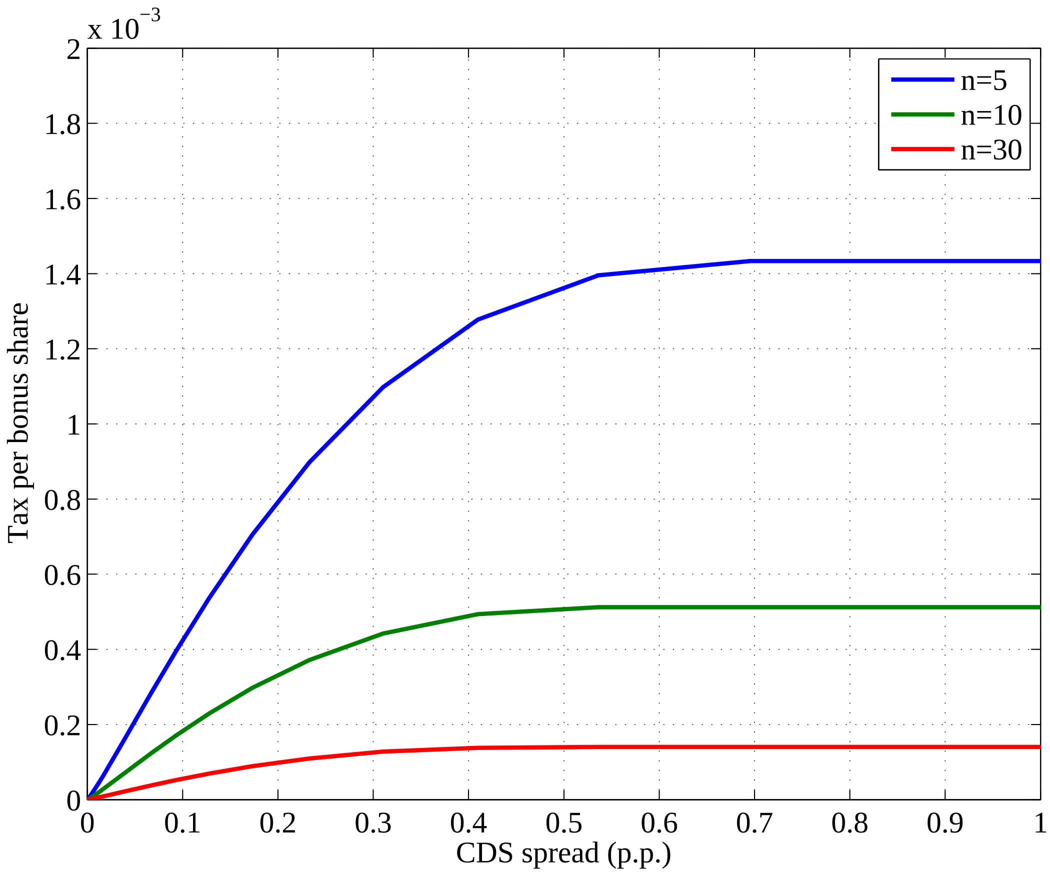


Figure 8: Optimal tax schedule as a function of CDS spread (II)

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